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Multidimensional Discrete Transformations and Their Applications to Discrete Equations

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Abstract. Some classes of discrete equations in multidimensional space are considered. In certain cases a general solution is constructed for such equations using special discrete transformations. Applying additional conditions one can extract the unique solution.

Keywords: Digital Pseudo-differential Equation, Periodic Wave Factorization, Solvability

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Оригинальное исследование

Многомерные дискретные преобразования и их применение к дискретным уравнениям

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Аннотация. Рассматриваются некоторые классы дискретных уравнений в многомерном пространстве. В некоторых случаях для таких уравнений строится общее решение с помощью специальных дискретных преобразований. Применяя дополнительные условия, можно извлечь единственное решение.

Ключевые слова: дискретное псевдодифференциальное уравнение, периодическая волновая факторизация, разрешимость

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1. Introduction

1.1. This paper is devoted to studying certain class of discrete equations. these equations are operator equations, and generating operators were called digital pseudo-differential operators [1, 2]. We are interested in solvability of such equations and their approximation properties for solving continuous ones. Existing methods [3, 4, 5] are developed for partial differential equations and related boundary value problems, but these methods are not appropriate for pseudo-differential equations. Latter equations present more general class of operator equations, and there is not complete theory for such equations, particularly in domains with a non-smooth boundary. Moreover, we think the discrete theory of pseudo-differential equations will be useful in digital signal processing [6, 7, 8] since functions of a discrete variable are models of signals and images.

Starting step, we work with model operators and canonical domains (cones) and use periodic analogue of the wave factorization [9] to describe solvability picture for discrete equations. this paper is related to a conical domain of a special type. A half-space case was considered earlier [1, 2]. The section below includes main notations and definitions. Let us remind that continuous half-space case was studied in details in [10].

1.2. We use the following notations. Let $\mathbb{Z}^3$ be the integer lattice in $\mathbb{R}^3$, $C_n = \{x \in \mathbb{R}^3 : x = (x_1, x_2, x_3), x_3 > a_n|x_1| + b_n|x_2|,\ a, b > 0\}$ be the four-faced angle, $C_{n,d} = h\mathbb{Z}^3 \cap C_n, h > 0$, $\mathbb{T} = [-\pi, \pi]$, $\hbar = h^{-1}$, and a_n, b_n can take values $n,\ 1/n,\ n \in \mathbb{N}$. We denote $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \in h\mathbb{Z}^3$ and consider functions of discrete variable $u_d(\tilde{x})$.

Let us denote

$$\zeta^2 = \sum_{k=1}^{3} \zeta_k^2, \quad \zeta_k = \hbar(e^{ih\cdot\xi_k} - 1),$$

and let $S(h\mathbb{Z}^3)$ be the discrete analogue of the Schwartz space of infinitely differentiable rapidly decreasing at infinity functions [1].

The space $H^s(h\mathbb{Z}^3)$ consists of discrete functions and it is a closure of the space $S(h\mathbb{Z}^3)$ with respect to the norm

$$||u_d||_s = \left(\int\limits_{\hbar\mathbb{T}^3} (1+|\zeta^2|)^s |\tilde{u}_d(\xi)|^2 d\xi\right)^{1/2},$$

where $\tilde{u}_d(\xi)$ denotes the discrete Fourier transform

$$(F_d u_d)(\xi) \equiv \tilde{u}_d(\xi) = \sum_{\tilde{x}\in h\mathbb{Z}^3} e^{i\tilde{x}\cdot\xi} u_d(\tilde{x}) h^3, \quad \xi \in \hbar\mathbb{T}^3.$$

Let $A_d(\xi)$ be a measurable periodic function defined in $\mathbb{R}^3$ with the basic cube of periods $\hbar\mathbb{T}^3$.

A digital pseudo-differential operator A_d with the symbol $A_d(\xi)$ in discrete cone $C_{n.d}$ is called the following operator

$$(A_d u_d)(\tilde{x}) = \sum_{\tilde{y}\in h\mathbb{Z}^3} h^3 \int\limits_{\hbar\mathbb{T}^3} A_d(\xi) e^{i(\tilde{y}-\tilde{x})\cdot\xi} \tilde{u}_d(\xi) d\xi, \quad \tilde{x} \in C_{n,d},$$

Here we will consider symbols satisfying the condition

$$c_1(1+|\zeta^2|)^{\alpha/2} \le |A_d(\xi)| \le c_2(1+|\zeta^2|)^{\alpha/2}$$

with positive constants c_1, c_2 non-depending on h. The number $\alpha \in \mathbb{R}$ is called an order of the digital pseudo-differential operator A_d.

We study solvability of the discrete equation

$$(A_d u_d)(\tilde{x}) = 0, \quad \tilde{x} \in C_{n,d}, \tag{1}$$

in the space $H^s(C_{n,d})$, it consists of functions from the space $H^s(h\mathbb{Z}^3)$ with supports in $\overline{C_{n,d}}$. For this purpose we need certain specific domains of three-dimensional complex space $\mathbb{C}^3$. A domain of the type $\mathcal{T}_h(C_n) = \hbar\mathbb{T}^3 + iC_n$ is called a tube domain over the cone C_n. Such domains are periodic analogues of radial tube domains [11, 12]. We will work with analytic functions $f(x+i\tau)$ in such domains $\mathcal{T}_h(C_n)$. Let us denote

$$\overset{*}{C}_n = \{x \in \mathbb{R}^3 : x_3 > \frac{1}{2a_n}|x_1| + \frac{1}{2b_n}|x_2|\},$$

it is so called conjugate cone to C_n.

Definition 1.1. *Periodic wave factorization of the symbol $A_d(\xi)$ with respect to C_n is called its representation in the form*

$$A_d(\xi) = A_{d,\neq}(\xi) A_{d,=}(\xi),$$

where factors $A_{d,\neq}(\xi), A_{d,=}(\xi)$ admit analytic continuation into tube domains $\mathcal{T}_h(\overset{*}{C}_n), \mathcal{T}_h(-\overset{*}{C}_n)$ respectively satisfying the estimates

$$c_1(1+|\hat{\zeta}^2|)^{\frac{æ}{2}} \le |A_{d,\neq}(\xi+i\tau)| \le c_1'(1+|\hat{\zeta}^2|)^{\frac{æ}{2}},$$

$$c_2(1+|\hat{\zeta}^2|)^{\frac{\alpha-æ}{2}} \le |A_{d,=}(\xi-i\tau)| \le c_2'(1+|\hat{\zeta}^2|)^{\frac{\alpha-æ}{2}},$$

with positive constants c_1, c_1', c_2, c_2' non-depending on h;

$$\hat{\zeta}^2 \equiv \hbar^2\left(\sum_{k=1}^{3}(e^{ih(\xi_k+i\tau_k)}-1)^2\right), \quad \xi = (\xi_1,\xi_2,\xi_3) \in \hbar\mathbb{T}^3,$$

$$\tau = (\tau_1,\tau_2,\tau_3) \in \overset{*}{C}_n.$$

The number $æ \in \mathbb{R}$ is called an index of periodic wave factorization.

2. Discrete Transformations and Transmutation Operators

2.1. Here we will discuss some discrete transformations and their periodic representation. These transformations have corresponding continuous analogues [13, 14]. Let us introduce the following transformation $T_{a_n,b_n} : h\mathbb{Z}^3 \to h\mathbb{Z}^3$ of the following type

$$\tilde{t}_1 = \tilde{x}_1$$
$$\tilde{t}_2 = \tilde{x}_2$$
$$\tilde{t}_3 = \tilde{x}_3 - a_n|\tilde{x}_1| - b_n|\tilde{x}_2|$$

Let $u_d \in S(h\mathbb{Z}^3)$. We would like to understand what is the discrete Fourier image of the function $T_{a_n,b_n}u_d$. We have

$$(F_d T_{a_n,b_n} u_d)(\xi) = \sum_{\tilde{x}\in h\mathbb{Z}^3} e^{i\tilde{x}\cdot\xi}(T_{a_n b_n} u_d)(\tilde{x})h^3 = \sum_{\tilde{x}\in h\mathbb{Z}^3} e^{i\tilde{x}\cdot\xi} u_d(\tilde{x}_1, \tilde{x}_2, \tilde{x}_2 - a_n|\tilde{x}_1| - b_n|\tilde{x}_2|)h^3$$

$$= \sum_{\tilde{t}\in h\mathbb{Z}^3} e^{i\tilde{t}_1\cdot\xi_1} e^{i\tilde{t}_2\cdot\xi_2} e^{i(\tilde{t}_3 + a_n|\tilde{t}_1| + b_n|\tilde{t}_2|)\cdot\xi_3} u_d(\tilde{t}_1, \tilde{t}_2 . \tilde{t}_3)h^3$$

$$= \sum_{\tilde{t}_1\in h\mathbb{Z}} e^{i\tilde{t}_1\xi_1 + a_n|\tilde{t}_1|\xi_3} h \left(\sum_{\tilde{t}_2\in h\mathbb{Z}} e^{i\tilde{t}_2\xi_2 + b_n|t_2|\xi_3} h \left(\sum_{\tilde{t}_3\in h\mathbb{Z}} e^{i\tilde{t}_3\xi_3} u_d(\tilde{t}_1, \tilde{t}_2, \tilde{t}_3)h \right)\right)$$

$$= \sum_{\tilde{t}_1\in h\mathbb{Z}} e^{i\tilde{t}_1\xi_1 + a_n|\tilde{t}_1|\xi_3} h \left(\sum_{\tilde{t}_2\in h\mathbb{Z}} e^{i\tilde{t}_2\xi_2 + b_n|\tilde{t}_2|\xi_3} \hat{u}_d(\tilde{t}_1, \tilde{t}_2, \xi_3)h \right),$$

where $\hat{u}_t(\tilde{t}_1, \tilde{t}_2, \xi_3)$ is the discrete Fourier transform on third variable.

Further, we need to calculate one-dimensional discrete Fourier transforms on variables $\tilde{t}_1, \tilde{t}_2$. We will use calculations from [2, 15]. Let $\mathbb{Z}_+\{0\} \cup \mathbb{N}$, $\mathbb{Z}_- = \mathbb{Z} \setminus \mathbb{Z}_+$, and $\chi_\pm$ be indicators of $\mathbb{Z}_\pm$. For functions $\varphi(x)$ of one variables defined in segment $[-\hbar\pi, \hbar\pi]$ the periodic analogue of Hilbert transform [13, 16, 17, 18, 19, 20] was introduced in [2, 15] in the following way

$$(H_h^{per}\varphi)(\xi) = \frac{h}{2\pi i} p.v. \int_{-\hbar\pi}^{\hbar\pi} \varphi(x) \cot \frac{h(x-\xi)}{2} dx,$$

where integral is meant in principal value (p.v.) sense. Two projectors P_h and Q_h are related to the operator H_h^{per}

$$P_h = \frac{1}{2}(I + H_h^{per}), \quad Q_h = \frac{1}{2}(I - H_h^{per}),$$

I is identity operator, so that the representation

$$\varphi = P_h\varphi + Q_h\varphi$$

is unique for arbitrary function $\varphi \in L^2[-\hbar\pi, \hbar\pi]$.

Moreover, if $\varphi_d(\tilde{x})$ is a function of discrete variable $\tilde{x} \in h\mathbb{Z}$ then

$$F_d(\chi_+ \cdot \varphi_d) = P_h(F_d\varphi_d), \quad F_d(\chi_- \cdot \varphi_d) = Q_h(F_d\varphi_d)$$

at least for $\varphi_d \in S(h\mathbb{Z})$.

Using these properties we introduce the periodic Hilbert transforms with a parameter of the following type $(\psi(\xi) = \psi(\xi_1, \xi_2, \xi_3))$

$$(H_h'\psi)(\xi) = \frac{h}{2\pi i} p.v. \int_{-\hbar\pi}^{\hbar\pi} \psi(\eta_1, \xi_2, \xi_3) \cot \frac{h(\eta_1 - \xi_1)}{2} d\eta_1,$$

$$(H_h''\psi)(\xi) = \frac{h}{2\pi i} p.v. \int_{-\hbar\pi}^{\hbar\pi} \psi(\xi_1, \eta_2, \xi_3) \cot \frac{h(\eta_2 - \xi_2)}{2} d\eta_2,$$

and corresponding projectors

$$P_h' = (I + H_h'), \quad Q_h' = (I - H_h'),$$

$$P_h'' = (I + H_h''), \quad Q_h'' = (I - H_h''),$$

and continue calculations for $F_d T_{a_n b_n} u_d$. We have

$$\sum_{\tilde{t}_2\in h\mathbb{Z}} e^{i\tilde{t}_2\xi_2 + b_n|\tilde{t}_2|\xi_3} \hat{u}_d(\tilde{t}_1, \tilde{t}_2, \xi_3)h =$$

$$\sum_{\tilde{t}_2\in h\mathbb{Z}_+} e^{i\tilde{t}_2\xi_2 + b_n\tilde{t}_2\xi_3} \hat{u}_d(\tilde{t}_1, \tilde{t}_2, \xi_3)h + \sum_{\tilde{t}_2\in h\mathbb{Z}_-} e^{i\tilde{t}_2\xi_2 - b_n\tilde{t}_2\xi_3} \hat{u}_d(\tilde{t}_1, \tilde{t}_2, \xi_3)h$$

$$= \sum_{\tilde{t}_2\in h\mathbb{Z}} e^{i\tilde{t}_2(\xi_2 + b_n\xi_3)} \chi_+(\tilde{t}_2)\hat{u}_d(\tilde{t}_1, \tilde{t}_2, \xi_3)h + \sum_{\tilde{t}_2\in h\mathbb{Z}} e^{i\tilde{t}_2(\xi_2 - b_n|\xi_3)} \chi_-(\tilde{t}_2)\hat{u}_d(\tilde{t}_1, \tilde{t}_2, \xi_3)h$$

$$= P''_h \hat{u}_d(\tilde{t}_1, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \hat{u}_d(\tilde{t}_1, \xi_2 - b_n\xi_3, \xi_3),$$

where $\hat{u}_d$ is the discrete Fourier of discrete function $u_d(\tilde{t}_1, \tilde{t}_2, \tilde{t}_3)$ on variables $\tilde{t}_2, \tilde{t}_3$.

Further we find

$$(F_d T_{a_n,b_n} u_d)(\xi) = \sum_{\tilde{t}_1 \in h\mathbb{Z}} e^{i\tilde{t}_1\xi_1 + a_n|t_1|\xi_3} \left(P''_h \hat{u}_d(\tilde{t}_1, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \hat{u}_d(\tilde{t}_1, \xi_2 - b_n\xi_3, \xi_3)\right) h$$

$$= \sum_{\tilde{t}_1 \in h\mathbb{Z}_+} e^{i\tilde{t}_1(\xi_1 + a_n\xi_3)} \left(P''_h \hat{u}_d(\tilde{t}_1, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \hat{u}_d(\tilde{t}_1, \xi_2 - b_n\xi_3, \xi_3)\right) h$$

$$+ \sum_{\tilde{t}_1 \in h\mathbb{Z}_-} e^{i\tilde{t}_1(\xi_1 - a_n\xi_3)} \left(P''_h \hat{u}_d(\tilde{t}_1, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \hat{u}_d(\tilde{t}_1, \xi_2 - b_n\xi_3, \xi_3)\right) h$$

$$= \sum_{\tilde{t}_1 \in h\mathbb{Z}} e^{i\tilde{t}_1(\xi_1 + a_n\xi_3)} \chi_+(\tilde{t}_1) \left(P''_h \hat{u}_d(\tilde{t}_1, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \hat{u}_d(\tilde{t}_1, \xi_2 - b_n\xi_3, \xi_3)\right) h$$

$$+ \sum_{\tilde{t}_1 \in h\mathbb{Z}} e^{i\tilde{t}_1(\xi_1 - a_n\xi_3)} \chi_-(\tilde{t}_1) \left(P''_h \hat{u}_d(\tilde{t}_1, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \hat{u}_d(\tilde{t}_1, \xi_2 - b_n\xi_3, \xi_3)\right) h$$

$$= P'_h \left(P''_h \tilde{u}_d(\xi_1 + a_n\xi_3, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \tilde{u}_d(\xi_1 + a_n\xi_3, \xi_2 - b_n\xi_3, \xi_3)\right)$$

$$+ Q'_h \left(P''_h \tilde{u}_d(\xi_1 - a_n\xi_3, \xi_2 + b_n\xi_3, \xi_3) + Q''_h \tilde{u}_d(\xi_1 - a_n\xi_3, \xi_2 - b_n\xi_3, \xi_3)\right).$$

So, we have the following operator acting on Fourier images in the following way

$$(V_{a_n,b_n} \tilde{u}_d)(\xi) = (P'_h P''_h \tilde{u}_d)(\xi_1 + a_n\xi_3, \xi_2 + b_n\xi_3, \xi_3) + (P'_h Q''_h \tilde{u}_d)(\xi_1 + a_n\xi_3, \xi_2 - b_n\xi_3, \xi_3)$$

$$+ (Q'_h P''_h \tilde{u}_d)(\xi_1 - a_n\xi_3, \xi_2 + b_n\xi_3, \xi_3) + (Q'_h Q''_h \tilde{u}_d)(\xi_1 - a_n\xi_3, \xi_2 - b_n\xi_3, \xi_3),$$

and finally

$$V_{a_n,b_n} = F_d T_{a_n,b_n} F_d^{-1}. \tag{2}$$

The discrete Fourier transform takes part in this relation as a transmutation operator.

2.2. Here we return to studying the equation (1).

Theorem 2.1. *Let the symbol $A_d(\xi)$ admits the periodic wave factorization with respect to C_n with the index æ such that $æ - s = n + \varepsilon$, $n \in \mathbb{N}$, $|\varepsilon| < 1/2$ then a general solution of the equation* (1) *has the following form*

$$\tilde{u}_d(\xi) = A^{-1}_{d,\neq}(\xi) V^{-1}_{a_n,b_n} \left(\sum_{k=0}^{n-1} \tilde{c}_{d,k}(\xi') \zeta_3^k \right), \tag{3}$$

where $\xi' = (\xi_1, \xi_2) \in \hbar^2\mathbb{T}^2$, $c_{d,k} \in H^{s_k}(h\mathbb{Z}^2)$ are arbitrary functions, $s_k = s - æ + k - 1/2$, $k = 0, 1, \dots, n-1$.

Proof. Let us introduce the discrete function v_d such that

$$v_d(\tilde{x}) = \begin{cases} -(A_d u_d)(\tilde{x}), & \tilde{x} \notin M_d, \\ 0, & \tilde{x} \in M_d. \end{cases}$$

Then we have the following paired equation in whole $h\mathbb{Z}^3$

$$(A_d u_d)(\tilde{x}) + v_d(\tilde{x}) = 0, \quad \tilde{x} \in h\mathbb{Z}^3. \tag{4}$$

Applying the discrete Fourier transform to (4) we have

$$A_d(\xi)\tilde{u}_d(\xi) + \tilde{v}_d(\xi) = 0,$$

and after periodic wave factorization it leads to the equality

$$A_{d,\neq} \tilde{u}_d(\xi) = -A^{-1}_{d,=} \tilde{v}_d(\xi),$$

so that after applying the inverse discrete Fourier transform we have

$$F_d^{-1} A_{d,\neq} \tilde{u}_d(\xi) = -F_d^{-1} A^{-1}_{d,=} \tilde{v}_d(\xi).$$

Now we apply the transformation $T_{a_n b_n}$ in the latter equality and obtain the following

$$T_{a_n b_n} F_d^{-1} A_{d,\neq} \tilde{u}_d(\xi) = -T_{a_n b_n} F_d^{-1} A^{-1}_{d,=} \tilde{v}_d(\xi). \tag{5}$$

We denote $\mathbb{Z}^3_\pm = \{\tilde{x} \in \mathbb{Z}^3 : \tilde{x} - (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3), \pm\tilde{x}_3 > 0\}$ and then the left hand side of (5) vanishes in $h\mathbb{Z}^3_-$, and the right hand side vanishes in $h\mathbb{Z}^3_+$. According to [1] (Theorem 2) such discrete function should be supported on discrete hyper-plane $\tilde{x}_3 = 0$, and its discrete Fourier transform looks as follows

$$F_d T_{a_n b_n} F_d^{-1} A_{d,\neq} \tilde{u}_d(\xi) = \sum_{k=0}^{n-1} \tilde{c}_{d,k}(\xi') \zeta_3^k,$$

where $\xi' = (\xi_1, \xi_2) \in \hbar^2\mathbb{T}^2$, $c_{d,k} \in H^{s_k}(h\mathbb{Z}^2)$ are arbitrary functions, $s_k = s - æ + k - 1/2$, $k = 0, 1, \ldots, n-1$.

Taking into account (2) we have the formula for a general solution of the equation (1)

$$\tilde{u}_d(\xi) = A^{-1}_{d,\neq}(\xi) V^{-1}_{a_n,b_n} \left(\sum_{k=0}^{n-1} \tilde{c}_{d,k}(\xi') \zeta_3^k \right),$$

Q.E.D.

Remark 2.1. *Obviously, the operator $V^{-1}_{a_n,b_n}$ is very simple,*

$$V^{-1}_{a_n,b_n} = V_{-a_n,-b_n}.$$

3. Discrete Boundary Value Problems

3.1. Here we consider a special case in which we can suggest simple solution. Namely, we assume that under conditions of Theorem 2.1 we have $æ - s = 1 + \varepsilon$, $|\varepsilon| < 1/2$. Then the formula (3) looks as follows

$$\tilde{u}_d(\xi) = A^{-1}_{d,\neq}(\xi) V^{-1}_{a_n,b_n} \tilde{c}_d(\xi'), \tag{6}$$

where we have written c_d instead of $c_{d,0}$ for simplicity.

To determine uniquely the function c_d we add the following condition

$$\sum_{\tilde{x}_3 \in h\mathbb{Z}_+} u_d(\tilde{x}', \tilde{x}_3) h = f_d(\tilde{x}'), \tag{7}$$

where f is given function.

Theorem 3.1. If conditions of Theorem 2.1 hold and $1/2 < æ - s < 3/2$, $f_d \in H^{s+1/2}(h\mathbb{Z}^2)$ then the problem (1),(7) has unique solution given by the formula

$$\tilde{u}_d(\xi) = A^{-1}_{d,\neq}(\xi) V^{-1}_{a_n,b_n} \tilde{c}_d(\xi'),$$

where $\tilde{c}_d$ is defined by the formula

$$\tilde{c}_d(\xi') = A_{\neq}(\xi', 0) \tilde{f}_d(\xi').$$

Proof. First we write the condition (7) in Fourier images

$$\tilde{u}_d(\xi_1, \xi_2, 0) = \tilde{f}_d(\xi_1, \xi_2). \tag{8}$$

Second we have

$$\begin{aligned}(V_{-a_n,-b_n}\tilde{c}_d)(\xi) &= (P'_h P''_h \tilde{c}_d)(\xi_1 - a_n\xi_3, \xi_2 - b_n\xi_3) + (P'_h Q''_h \tilde{c}_d)(\xi_1 - a_n\xi_3, \xi_2 + b_n\xi_3) \\ &+ (Q'_h P''_h \tilde{c}_d)(\xi_1 + a_n\xi_3, \xi_2 - b_n\xi_3) + (Q'_h Q''_h \tilde{c}_d)(\xi_1 + a_n\xi_3, \xi_2 + b_n\xi_3).\end{aligned} \tag{9}$$

Substituting $\xi_3 = 0$ in the formula (9) we obtain

$$\begin{aligned}(V_{-a_n,-b_n}\tilde{c}_d)(\xi_1, \xi_2, 0) &= (P'_h P''_h \tilde{c}_d)(\xi_1, \xi_2) + (P'_h Q''_h \tilde{c}_d)(\xi_1, \xi_2) \\ &+ (Q'_h P''_h \tilde{c}_d)(\xi_1, \xi_2) + (Q'_h Q''_h \tilde{c}_d)(\xi_1, \xi_2).\end{aligned}$$

Taking into account properties of projectors P'_h, P''_h, Q'_h, Q''_h we find

$$(V_{-a_n,-b_n}\tilde{c}_d)(\xi_1, \xi_2, 0) = \tilde{c}_d(\xi_1, \xi_2).$$

According to formula (6) we have

$$\tilde{u}_d(\xi', 0) = A^{-1}_{\neq}(\xi', 0) \tilde{c}_d(\xi')$$

and then using (8) we conclude

$$\tilde{c}_d(\xi') = A_{\neq}(\xi', 0) \tilde{f}_d(\xi').$$

Q.E.D.

Remark 3.1. *The continuous analogue of the problem* (1),(7) *was considered in [21]. Unique solvability for such problem was proved under corresponding assumptions.*

Conclusion. The considered discrete boundary value problem (1),(7) should be approximation problem for corresponding continuous boundary value problem. Here the first step was done, the unique solvability and integral representation were obtained. The next step is a comparison (in certain sense) of discrete and continuous solutions and we will give it in forthcoming papers. In two-dimensional case such a comparison was obtained.

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